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Fractional Hartley Transform on G-Boehmian Space

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ABSTRACT: Using a special type of fractional convolution, a G-Boehmian space $\mathcal{B}_\alpha$ containing integrable functions on $\mathbb{R}$ is constructed. The fractional Hartley transform (FRHT) is defined as a linear, continuous injection from $\mathcal{B}_\alpha$ into the space of all continuous functions on $\mathbb{R}$. This extension simultaneously generalizes the fractional Hartley transform on $L^1(\mathbb{R})$ as well as Hartley transform on an integrable Boehmian space.

Key Words: Fractional Hartley transform, Fractional convolution, Boehmians.

Contents

1. Introduction

Hartley introduced a Fourier-like transform in 1942, which is called Hartley transform (see [7,11]). Like the fractional Fourier transform (FRFT) [21], many integral transforms have been generalized to the corresponding fractional integral transforms. In particular, fractional Fourier cosine transform (FRFCT), fractional Fourier sine transform (FRFST) and fractional Hartley transform (FRHT) were defined and used extensively in signal processing [4,24].

In [14], Mikusiński, J. and Mikusiński, P., introduced Boehmian space, which in general, consists of convolution quotients of sequences of functions. In [15], an abstract Boehmian space $\mathcal{B}$ is constructed by using a complex topological vector space G, $S \subset G$, $\star : G \times S \to G$ and a collection Δ of sequences satisfying certain axioms. As many of these Boehmian spaces contain the respective domains of various classical integral transforms, the research on Boehmian space includes extension of integral transforms to larger domains. For example, we refer [1,2,3,6,29,5,9,10,12,22,23,25,26,27]. Meanwhile, various versions of Boehmian spaces are introduced with new assumptions or slightly weaker assumptions than that are used in the general construction of a Boehmian space given in [15], by many authors [8,13,17,18,19]. Most recently, the G-Boehmian space is introduced in [9] as a generalization of the Boehmian space and the Hartley transform is extended to a suitable G-Boehmian space. In the present article, we introduce a special type of fractional convolution to construct a G-Boehmian space $\mathcal{B}_\alpha$ containing the space of integrable functions on $\mathbb{R}$. The fractional Hartley transform (FRHT) is extended consistently as a linear, continuous injection from $\mathcal{B}_\alpha$ in to the space $C(\mathbb{R})$, of all complex-valued continuous functions on reals.

This paper is organized as follows. In Section 2, we recall fractional Hartley transform, the general construction of a G-Boehmian space and some of their properties. In Section 3, we shall prove all the preliminary results required for the construction of the G-Boehmian space $\mathcal{B}_\alpha$. In Section 4, we provide the extended FRHT on this G-Boehmian space and investigate its properties.

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