

The Non-Neighbor Harmonic Index on Elementary Graph Operations

A. Rizwana^{1*}, G. Jeyakumar²

¹Department of Mathematics, Sadakathullah Appa College, Manonmaniam Sundaranar University, Tirunelveli-12, India

² Department of Mathematics, St.John's College, Tirunelveli-2, India

*Corresponding Author: rijurizwana@gmail.com, Tel.: +91-8508435629

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Abstract—The aim of this paper is to study the behaviour of non-neighbor harmonic index of graphs with respect to the removal of pendant edge and an edge with maximal weight. The non-neighbor harmonic index for the subdivision graphs are computed and discussed in detail.

Keywords— non-neighbor harmonic index, elementary graph operations, subdivision graphs.

I-INTRODUCTION

To bring the power of mathematics to bear on real-world problems, the problem should be first modelled mathematically. Graphs, representatives of mathematics, are remarkable versatile tools for modelling. Graph theory is largely applied to the characterization of chemical structures, as well as to qualitative and quantitative structure-property (QSPR) and structure-activity (QSAR) relations by means of certain numerical characteristics, the topological indices [1]. A single number that can be used to characterize some property of the graph of a molecule is called a topological index. Many topological indices [2,3] have been introduced and studied: Randic index [4], Wiener index [5], First and Second Zagreb indices [6] are a few examples of these concepts. Many mathematical properties of these graph invariants have been studied.

In this paper we are concerned with simple graphs, having no directed or weighted edges, and no self-loops. A graph G is an ordered pair of two sets V and E . The set $V = V(G)$ is a finite non empty set and $E = E(G)$ is a binary relation defined on V . A graph can be visualized by representing the elements of V by vertices and joining the pair of vertices u, v by an edge if and only if $uv \in E(G)$. Also we denote $|V(G)| = n$ and $|E(G)| = m$. The degree of the vertex $v \in V(G)$, written $d(v)$, is the number of first neighbors of v in the underlying graph G .

Graph operations produce new graphs from initial ones. They may be classified into elementary operations or advanced operations. Elementary operations or editing operations create a new graph from initial one by a simple local change such as addition or deletion of a vertex or of an

edge, merging and splitting of vertices or edges. We define $G - uv$ to be the graph obtained from G by deleting the edge $uv \in E(G)$, and $G + uv$ to be the graph that arises from G by adding an edge uv between two non-adjacent vertices u and v of G . The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently by inserting an additional vertex into each edge of G . We use S_n , P_n and K_n to denote the star, path and complete graph on n vertices, respectively. For undefined terminology and notations in the paper, we refer to [7].

The paper is organized as follows, Section I contains the introduction of topological indices, graphs and elementary graph operations. Section II contain the related work of harmonic index and non-neighbor harmonic index, Section III presents the results and discussion of how the non-neighbor harmonic index of a graph G strictly increases and decreases by the removal of pendant edge [8] and an edge with maximal weight respectively. Also the non-neighbor harmonic index for the subdivision graph $S(G)$ is computed for some graphs and the results are discussed in detail. Section IV concludes the research work with the scope for future.

II-RELATED WORK

In the 1980's, Siemion Fajtlowicz created a vertex-degree-based quantity which was re-introduced by Zhong [9] in 2012 called Harmonic Index [10]. The harmonic index is one of the most important indices in chemical and mathematical fields. The harmonic index gives better correlations with physical and chemical properties of molecules. It is defined as