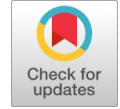


# The Radial Radio Number and the Clique Number of a Graph



Selvam Avadayappan, M. Bhuvaneshwari, S. Vimalajenifer

**Abstract:** Let  $G(V(G), E(G))$  be a graph. A radial radio labeling,  $f$ , of a connected graph  $G$  is an assignment of positive integers to the vertices satisfying the following condition:  $d(u, v) + |f(u) - f(v)| \geq 1 + r(G)$ , for any two distinct vertices  $u, v \in V(G)$ , where  $d(u, v)$  and  $r(G)$  denote the distance between the vertices  $u$  and  $v$  and the radius of the graph  $G$ , respectively. The span of a radial radio labeling  $f$  is the largest integer in the range of  $f$  and is denoted by  $span(f)$ . The radial radio number of  $G$ ,  $rr(G)$ , is the minimum span taken over all radial radio labelings of  $G$ . In this paper, we construct a graph  $A$  for which the difference between the radial radio number and the clique number is the given non negative integer.

**Keywords:** diameter, frequency assignment problem, radius, radio labeling, radio number, radial radio number, radial radio number. AMS Subject Classification Code(2010):05C78

## I. INTRODUCTION

In this paper, by a graph, we mean only finite, simple, undirected and connected graph. For basic notations and terminology, we follow [4]. Let  $G = (V(G), E(G))$  be a graph. The distance  $d(u, v)$  between any two vertices  $u$  and  $v$ , is the length of a shortest  $(u, v)$ - path in  $G$ . The eccentricity,  $e(u)$ , of a vertex  $u$  in  $V(G)$  is the distance of a vertex farthest from  $u$ . The radius of a graph  $G$  is the minimum eccentricity among all the vertices and is denoted by  $r(G)$  or  $r$ . The diameter of  $G$  is the maximum eccentricity among all the vertices and is denoted by  $diam(G)$  or  $d$ . The relation between  $r(G)$  and  $diam(G)$  is given by the inequality  $r(G) \leq diam(G) \leq 2r(G)$  [8]. For further details on distance in graphs, one can refer [5].

For a subset  $S$  of  $V(G)$ , let  $\langle S \rangle$  denote the induced subgraph of  $G$  induced by  $S$ . A clique  $C$  is a subset of  $V(G)$  with maximum number of vertices such that  $\langle C \rangle$  is complete. The clique number of a graph  $G$ ,

denoted by  $\omega(G)$  or  $\omega$ , is the number of vertices in a clique of  $G$ .

In 1960's Rosa[12] introduced the concept of graph labeling. A graph labeling is an assignment of numbers to the vertices or edges or both, satisfying some constraints. Rosa named the labeling introduced by him as  $\beta$ -valuation and later on it becomes a very famous interesting graph labeling called graceful labeling, which is the origin for any graph labeling problem. Motivated by the real life problems, many mathematicians introduced various labeling concepts[9]. Here, we see one of the familiar graph labelings in graph theory.

The problem of assigning frequencies to the channels for the FM radio stations is known as Frequency Assignment Problem (FAP). This problem was studied by W. K. Hale[10].

In a telecommunication system, the assignment of channels to FM radio stations play a vital role. Motivated by the FAP, Chartrand et al.[6] introduced the concept of radio labeling. For a given  $k$ ,  $1 \leq k \leq diam(G)$ , a radio  $k$ -coloring,  $f$ , is an assignment of positive integers to the vertices satisfying the following condition:

$$d(u, v) + |f(u) - f(v)| \geq 1 + k \quad (1)$$

for any two distinct vertices  $u, v \in V(G)$ . Whenever,  $diam(G) = k$ , the radio  $k$ - coloring is called a radio labeling[7] of  $G$ . The span of a radio labeling  $f$  is the largest integer in the range of  $f$  and is denoted by  $span(f)$ . The radio number of  $G$  is the minimum span taken over all radio labelings of  $G$  and is denoted by  $rn(G)$ . Motivated by the work of Chartrand et al., on radio labeling, KM. Kathiresan and S. Vimalajenifer[11] introduced the concept of radial radio labeling. A radial radio labeling  $f$  of  $G$  is a function  $f: V \rightarrow \{1, 2, \dots\}$  satisfying the condition,

$$d(u, v) + |f(u) - f(v)| \geq 1 + r(G) \quad (2)$$

for any two distinct vertices  $u, v \in V(G)$ . This condition is obtained by taking  $k = r(G)$  in (1). The above condition is known as radial radio condition. The span of a radial radio labeling  $f$  is the largest integer in the range of  $f$ . The radial radio number is the minimum span taken over all radial radio labelings of  $G$  and is denoted by  $rr(G)$ .

That is,  $rr(G) = \min_f \max_{v \in V(G)} f(v)$ , where the

minimum runs over all radial radio labelings of  $G$ .

Let  $f$  be a radial radio labeling of a graph  $G$  and let  $C$  be a clique in  $G$ .

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