

# Support Independence in Graphs

Selvam Avadayappan, M. Bhuvaneshwari and Rajeev Gandhi

Research Department of Mathematics  
VHNSN College, Virudhunagar – 626001, India.

## Abstract

In any graph  $G$ , the support of a vertex is defined as the sum of degrees of its neighbours. A graph  $G$  is said to be balanced, if every vertex of  $G$  has same support.  $G$  is called highly unbalanced when no two vertices of  $G$  have same support. In this paper, we introduce the concept of support independence in graphs. A subset  $S$  of a vertex set is said to be support independent, if no two vertices in  $S$  are having same support. The support independence number of  $G$  is the cardinality of maximum support independent set in  $G$ . We obtain the support independence number of some standard graphs and derived graphs.

**Keywords:** Splitting graphs, cosplitting graphs, Support of a vertex, Support independence.

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## 1 Introduction

Only finite, simple, undirected graphs are considered in this paper. We refer [7] for further notations and terminology. The degree of a vertex  $v$  is denoted by  $d(v)$ . A *full vertex* of  $G$  is a vertex which is adjacent to every other vertices of  $G$ . A graph  $G$  is said to be  $r$ -regular, if every vertex of  $G$  has degree  $r$ .  $D(G)$  denote the set of degrees of all vertices in  $G$ .

In a graph  $G(V, E)$ , for any vertex  $v \in V$ , the *open neighbourhood* of  $v$  is the set of all vertices adjacent to  $v$ . That is,  $N(v) = \{u \in V(G) / uv \in E(G)\}$ . The *closed neighbourhood* of  $v$  is defined by  $N[v] = N(v) \cup \{v\}$ . Clearly, if  $N[u] = N[v]$ , then  $u$  and  $v$  are adjacent and  $d(u) = d(v)$ .

The concept of support of a vertex has been introduced and studied by Selvam Avadayappan and G. Mahadevan [6]. The *supports* of a vertex  $v$  is the sum of degrees of its neighbours. That is,  $s(v) = \sum_{u \in N(v)} d(u)$ . Note that the support of any vertex in an  $r$ -regular graph is  $r^2$ .

A graph  $G$  is said to be a *balanced graph*, if any two vertices in  $G$  have the same support. It is easy to observe that the complete bipartite graphs  $K_{m,n}$  and any regular graphs are balanced graphs. A graph  $G$  is said to be *highly unbalanced*, if distinct vertices of  $G$  have distinct supports. For example, a highly unbalanced graph is shown in Figure 1.

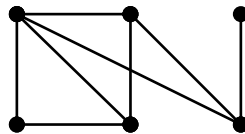


Figure 1